1. Solve for x in each of the following Quadratic equations:

(a) $15x^2 - 8 = 14x$

(b) $(x + 1)(x - 3) = 5$

(c) $\frac{x(x-5)}{6} - 1 = 0$

(d) $x^2 - x = 0$

(e) $x(x + 3) - 1 = 0$

(f) $2x^2 - 2x = 12$

(g) $x^2 - x - 12 = 0$

(h) $(x - 3)^2 - 3x(x - 2) = 0$

(i) $\frac{1}{2}x^2 + 3x - 10 = 0$

(j) $(x - 5)(x + 3) = 0$

(k) $x(x - 7) = 0$

(l) $x^2 - 5x + 6 = 0$

(m) $4x^2 - 5x = 3$

(n) $x^2 + 9x - 3 = 0$ $(Correct\ to\ TWO\ decimal\ places)$

(o) $x^2 - 5x - 2 = 0$ $(Correct\ to\ TWO\ decimal\ places)$

(p) $x^2 - 6x + 2 = 0$ $(Correct\ to\ TWO\ decimal\ places)$

(q) $2x^2 - 5x + 12 = 0$

(r) If a quadratic is in the form $ax^2 + bx + c = 0$, prove by completing the square that the roots can be found in the form : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

(s) $x^2 - 7 = 0$ as a difference of two squares.

2. Inequality equations

(a) $x^2 + 8x + 16 > 0$

(b) $x(x - 4) < 0$

(c) $x(4 - x) < 0$

(d) $(3 - x)(x + 1) > 0$

(e) $x^2 \geq 20 + x$

(f) $3(x + 7) < \frac{x}{2} + 1$

(g) Given $f(x) = \frac{2x^2 - x}{5}$ Determine the values of x for which f(x) ≥ 0

(h) $\frac{x^2}{x+2} \leq 0$

3. Exponential equations

Solve for x in each of the following:

(a) $5^x = \frac{1}{125}$

(b) $3^{x+3} - 3^{x+2} = 486$

(c) $4^x - 2^x = 2$

(d) $x^{-\frac{3}{4}} = 8$

(e) $2^{3x+1} + 2^x = 12$

(f) $2^{x\sqrt{x}} = 2^{27}$

(g) $3^{2x} - 7.3^x = 18$

(h) $x - 3x^{\frac{1}{2}} = 4$

(i) $3x^3 = 81$

Expressions and Equations with Surds

1. Solve for x in each of the following:

   (a) $\sqrt{2x+1} = x - 1$

   (b) $\sqrt{\frac{x}{2} + 3} = 4 - x$

   (c) $\sqrt{x-1} + 3 = x$

   (d) $\sqrt{x^2 - 5} = 2\sqrt{x}$

   (e) $9 = \sqrt{x}$

2. Simplify the following, without the use of a calculator:

   (a) Show that $\sqrt[12]{10} \times \sqrt[6]{640} \times \sqrt[4]{810} \times \sqrt{40} = 120$

   (b) $\sqrt{72x^2} - \sqrt{98x^2} + 2\sqrt{288x^2}$

   (c) $\sqrt{x + \sqrt{2x-1}}.\sqrt{x - \sqrt{2x-1}}$

   (d) if $x = \frac{3-\sqrt{a}}{\sqrt{2}}$ and $y = \frac{4+\sqrt{a}}{\sqrt{2}}$ Deterrmine the value of $(x+y)^2$

   (e) Show that $\frac{9-\sqrt{54}}{6\sqrt{2}} = \frac{3\sqrt{2}-2\sqrt{3}}{4}$

Solving simultaneous equations

1. Solve for x and y simultaneously:

(a) $4x^2 - y^2 = 171 \; and \; 2x - y = 9$

(b) $x^2 - xy + 3y^2 = 15 \; and \; y + 7 = 2x$

(c) $x^2 - 3x - 4 - y = y^2 \; and \; 2x - y + 1 = 0$

(d) $x - 2y = -3 \; and \; xy = 20$

(e) $2^{3x+1} = 4^y \; and \; x^2 + 2y = 29$

(f) $x = 2y \; and \; x^2 - 5xy = -24$

(g) $(2x - y)^2 + (x - 3)^2$

(h) $y = \frac{5}{x-2} \; and \; y + 1 = 2x$

(i) $y = \frac{2x+2}{4} \; and \; y - 2x^2 + 3x + 5 = 2x - 2x$

(j) $xy = 12 \; and \; 7 = x + y$

(k) $6 - 4x - y = 0 \; and \; 12 - 2x^2 - y = 0$

(l) $1 - x + y = 0 \; and \; y = 6 - 5x + x^2$

Nature of roots

1. For which value of k will the roots of $6x^2 + 6 = 4kx$ be real and equal.

2. Show that the roots of $(x + h)(x + k) = 4d^2$ are real for all values of $h, k\ and\ d$.

3. Show that $k^2x^2 + 2 = kx - x^2$has non-real roots for all real values of k.

4. The equation $x^2 + 12x = 3kx^2 + 2$ has real roots.

   (a) Find the greatest value of k such that $k \in \mathbb{Z}$.

   (b) Find the rational value of k for which the above equation has rational roots.

5. Consider the equation $k = \frac{x^2-4}{2x-5}$.where $x \neq \frac{5}{2}$

   (a) Find the value of k for which the roots are equal.

   (b) Find an integer k for which the roots of the equation will be rational or unequal.

6. (a) Prove that the roots of the equation $x^2 - (a + b)x + ab - p^2 = 0$ are real for all values of $a, b\ and\ p$.

   (b) When will the roots of the equation be equal?